PROBING — *Continued from page 12*

Peirce was suffering from a profound spiritual crisis, which was, at least partially, resolved two years later (April 24, 1892) by a mystical experience, of which he said afterwards (1898) "If . . . a man has had no religious experience, then any religion not an affectation is as yet impossible for him; and the only worthy course is to wait quietly till such experience *comes.* No amount of speculation can take place of experience" (1.655). This account is not one of Cartesian intuition. The idea that the practice of philosophy is not the place to look for "new truth" is not the view of Peirce, for whom abduction, the beginning of philosophy, is the only way to originate or advance knowledge.

NEW QUESTIONS

Question 14.

On several occasions Peirce stresses that we should not underestimate the power of science. In an article written for *The Christian Register,* "On Science and Immortality" (reprinted as CP 6.548–556), he writes: "The history of science affords illustrations enough of the folly of saying that this, that, or the other can never be found out. . . . Legendre said of a certain proposition in the theory of numbers that, while it appeared to be true, it was most likely beyond the powers of the human mind to prove it; yet the next writer on the subject gave six independent demonstrations of the theorem." (CP 6.446, 1887). Shortly after, Peirce repeats the same point in the unpublished "Reflections on the Logic of Science," where he again cites Legendre as an example: "Legendre in his *Théorie des nombres,* after penetrating more deeply into the higher arithmetic, and after having studied the nature of mathematical proof more accurately than any man before him had ever done, gave it as his opinion that the demonstration of a certain proposition,—though the proposition itself seemed to be true,— was probably beyond the powers of the human mind. Yet the very next important book on the subject published a few years later gave six proofs of this theorem, resting upon as many different principles" (MS 246. 4:1889).

Now here is the problem. The proposition in question is most likely the law of quadratic reciprocity of which Carl Gauss provides six proofs in his 1801 *Disquisitiones arithmeticae.* This proposition first surfaces in Legendre's "Recherches d'Analyse Indéterminée," which appeared in *Hist. Acad. Roy. des Sciences,* 1785, pp. 513–17. According to its English translation, however, Legendre's claim is a far cry from the extreme statement ascribed to him by Peirce. Legendre simply writes that the proposition is "quite difficult to prove," and that "I content myself with outlining the means for proving the theorem" (see *The History of Mathematics: A Reader,* edited by John Fauvel and Jeremy Gray. New York: Macmillan Press, 1988, p. 500). The translation not only suggests that Legendre believed that the proposition was provable, but also that he had at least some idea of how to prove it.

To make matters even more interesting, there are good indications that Legendre actually formulated the proof well before Gauss did, perhaps even in his 1785 paper. So far we have only been able to examine the 1808 edition of the *Essai sur la Théorie des Nombres.* There Legendre writes in the "Avertissement" to the second edition, that the proof of the law of quadratic reciprocity is slightly perfected (a été perfectionnée à quelques égards), clearly suggesting that the proof has been given in the first edition. On the next page Legendre notes that much of what he wrote in the first edition finds a close analogue in Gauss' *Disquisitiones,* including a "direct and very ingenious demonstration" of the law of reciprocity, which he includes in the new edition. Moreover, in the reprinted preface to the first edition, Legendre refers to his 1785 paper, noting as one of its three main accomplishments the demonstration of the law of reciprocity: "la démonstration d'une loi générale qui existe entre deux nombres premiers quelconques, et qu'on peut appeler *loi de réciprocité.*"

This account, contra Peirce's, is confirmed by W. W. Rouse Ball (*A Short Account of the History of Mathematics,* 4th ed., 1908, pp. 423–24), who writes: "The law of quadratic reciprocity, which connects any two odd primes, was first proved in this book *Théorie des Nombres,* but the result had been enunciated in a memoir of 1785 'Recherches d'Analyse Indéterminée.' Gauss called the proposition 'the gem of arithmetic,' and no less than six separate proofs are to be found in his works."

This leaves us with the following questions. First, did Legendre indeed claim at one point that proving the theorem was beyond the powers of the human mind, as Peirce claims he did? It might be that some of the force of his language got lost in the translation. Admittedly, on page 393 of the second edition, Legendre does speak of "almost insurmountable difficulties" ("des difficultés presqu'insurmontables"). Second, who was the first to provide the proof? Third, was there a persistent rumor, still very much alive in the 19th century, that Legendre made the claim Peirce ascribes to him? Perhaps Legendre made the claim when he was still a young man, and that he proved himself wrong in 1785. Or is this a case of a mistaken identity and is the statement made by another mathematician around this time? We would also be interested in photocopies of Legendre's 1785 paper and of the relevant section of the first edition of the *Essai sur la Théorie des Nombres,* as we have not yet been able to lay hands on these.

Question 15.

In a short piece entitled "Notes on the Question of the Existence of an External World" (in MS 971), Peirce makes a reference to W.K. Clifford. Peirce writes the following:

"But what evidence is there that we can immediately know only what is 'present' to the mind? The idealists generally treat this as self-evident; but, as Clifford jestingly says, 'It is evident' is a phrase which only means 'we do not know how to prove.'"

Can someone help us identify the source of this quotation?

RESEARCH GROUP ON SEMIOTIC EPISTEMOLOGY AND MATHEMATICS EDUCATION

University of Bielefeld

The Research Group is a part of the Institut für Didaktik der Mathematik at the University of Bielefeld. It studies the development of knowledge in historical and epistemological perspectives. The main interest is the relation between social and object-centered aspects of learning processes. One important thesis is that the process of learning mathematics can be used as a paradigm for discussing major problems of epistemology. The theoretical framework is provided by the philosophy of Charles S. Peirce and, in particular, by his considerations on the concept of sign, the process of generalization, and the role of continuity within the latter. The following projects are in progress. (1) Learning as a process of generalization (Michael Otte, Michael Hoffmann). (2) Peirce's philosophy of mathematics in the context of his evolutionary realism. The Peircean principle of continuity (Otte, Hoffmann). With respect to the philosophy of mathematics, the thesis is that Peirce's emphasis on the reality of generals, together with his semiotic model of the processuality of generalization, offers the possibility for a mathematical realism which is not reducible to the distinction of logicism, formalism, and intuitionism. And with respect to philosophy, the thesis is that the Peircean approach to the mathematical process of generalization can be understood as a paradigm which may be of special interest for problems of epistemology, ontology, and the development of social communities. Insofar as the concepts of processuality and evolution are based on the possibility of continuity, a main problem is the role of the concept of continuity in Peirce's philosophy. (3) The symmetry of subjectivity and objectivity in scientific generalization. Studies concerning the foundation of scientific rationality in the mathematical philosophy of Charles S. Peirce and his followers (Otte, Thomas Mies, Hoffmann). (4) Didactical aspects in Wittgenstein's philosophy of mathematics (Norbert Meder). (5) The Axiomatization of Arithmetic (Mircea Radu). (6) The interdependence of logic, ethics and aesthetics (Otte, Hoffmann).

For more information see http://www.uni-bielefeld.de/idm/arbeit/agsem.htm or contact Prof. Dr. Michael Otte or Dr. Michael Hoffmann, Institut für Didaktik der Mathematik, Universität Bielefeld, Postfach 100131, D-33501 Bielefeld. E-mail: michael.otte@post.uni-bielefeld.de, or: michael.hoffmann@post.uni-bielefeld.de.